

# Number Sense and Numeracy: Fractions

## Example 1

Add:  $\frac{7}{10} + \frac{2}{15}$

### Solution

$$\begin{aligned} \frac{7}{10} + \frac{2}{15} &= \frac{21}{30} + \frac{4}{30} \\ &= \frac{25}{30} \\ &= \frac{5}{6} \end{aligned}$$

Find a common denominator.  
Express in lowest terms.

## Example 2

Subtract:  $2\frac{1}{4} - \frac{1}{2}$

### Solution

$$\begin{aligned} 2\frac{1}{4} - \frac{1}{2} &= 2\frac{1}{4} - \frac{2}{4} \\ &= 1\frac{5}{4} - \frac{2}{4} \\ &= 1\frac{3}{4} \end{aligned}$$

Regroup.

## Example 3

Multiply:  $\frac{3}{5} \times 1\frac{1}{9}$

### Solution

$$\begin{aligned} \frac{3}{5} \times 1\frac{1}{9} &= \frac{3}{5} \times \frac{10}{9} \\ &= \frac{1\cancel{3}}{5} \times \frac{10}{\cancel{9}^2} \\ &= \frac{2}{3} \end{aligned}$$

Express in lowest terms. Then multiply.

## Example 4

Divide  $1\frac{2}{3} \div \frac{3}{10}$

### Solution

$$\begin{aligned} 1\frac{2}{3} \div \frac{3}{10} &= \frac{5}{3} \div \frac{3}{10} \\ &= \frac{5}{3} \times \frac{10}{3} \\ &= \frac{50}{9} \\ &= 5\frac{5}{9} \end{aligned}$$

Multiply by the reciprocal.

## Practise

1. Write the missing information to form equivalent fractions.

(a)  $\frac{1}{3} = \frac{\blacksquare}{18}$     (b)  $\frac{\blacksquare}{36} = \frac{1}{9}$     (c)  $\frac{\blacksquare}{28} = \frac{4}{7}$   
 (d)  $\frac{1}{5} = \frac{7}{\blacksquare}$     (e)  $\frac{3}{8} = \frac{15}{\blacksquare}$     (f)  $\frac{18}{\blacksquare} = \frac{2}{9}$   
 (g)  $\frac{1}{\blacksquare} = \frac{9}{36}$     (h)  $\frac{3}{\blacksquare} = \frac{15}{55}$     (i)  $\frac{5}{35} = \frac{\blacksquare}{7}$

2. Add.

(a)  $\frac{1}{7} + \frac{3}{7}$     (b)  $\frac{2}{9} + \frac{5}{9}$     (c)  $\frac{3}{8} + \frac{1}{8}$   
 (d)  $\frac{1}{3} + \frac{1}{9}$     (e)  $\frac{1}{3} + \frac{1}{6}$     (f)  $\frac{1}{3} + \frac{5}{12}$

3. Subtract.

(a)  $\frac{5}{9} - \frac{1}{9}$     (b)  $\frac{14}{15} - \frac{7}{15}$     (c)  $\frac{7}{15} - \frac{2}{5}$   
 (d)  $\frac{5}{6} - \frac{3}{8}$     (e)  $\frac{3}{4} - \frac{1}{6}$     (f)  $\frac{1}{3} - \frac{1}{6}$

4. Add.

(a)  $1\frac{1}{6} + 2\frac{1}{6}$     (b)  $2\frac{3}{10} + 1\frac{3}{10}$   
 (c)  $3\frac{3}{4} + 1\frac{3}{4}$     (d)  $1\frac{3}{4} + 2\frac{5}{12}$   
 (e)  $2\frac{1}{6} + 1\frac{5}{6}$     (f)  $4\frac{2}{5} + 3\frac{1}{5}$

5. Subtract.

(a)  $3\frac{5}{9} - 1\frac{2}{9}$     (b)  $4\frac{7}{16} - 2\frac{3}{16}$   
 (c)  $3\frac{3}{10} - 1\frac{7}{10}$     (d)  $2 - 1\frac{1}{4}$   
 (e)  $5\frac{1}{11} - 4\frac{9}{11}$     (f)  $2\frac{3}{7} - 1\frac{6}{7}$

6. Multiply.

(a)  $\frac{1}{2} \times \frac{3}{5}$     (b)  $\frac{3}{4} \times \frac{7}{10}$   
 (c)  $\frac{3}{5} \times 15$     (d)  $\frac{2}{3} \times \frac{9}{11}$   
 (e)  $\frac{3}{4} \times \frac{8}{15}$     (f)  $2\frac{1}{3} \times \frac{3}{14}$

7. Divide.

(a)  $\frac{3}{7} \div \frac{4}{5}$     (b)  $\frac{2}{11} \div \frac{3}{5}$   
 (c)  $\frac{3}{4} \div \frac{7}{8}$     (d)  $\frac{5}{8} \div \frac{13}{16}$   
 (e)  $2 \div \frac{2}{3}$     (f)  $4 \div \frac{8}{9}$   
 (g)  $\frac{3}{4} \div 9$     (h)  $\frac{5}{7} \div 10$

8. Arrange the fractions in order from least to greatest in value.

(a)  $\frac{3}{4}, \frac{5}{8}, \frac{1}{2}$     (b)  $\frac{7}{8}, \frac{3}{4}, \frac{13}{16}$   
 (c)  $\frac{3}{5}, \frac{9}{10}, \frac{3}{4}$     (d)  $\frac{5}{6}, \frac{8}{9}, \frac{2}{3}$

## Number Sense and Numeracy: Fractions

1. (a) 6    (b) 4    (c) 16  
 (d) 35    (e) 40    (f) 81  
 (g) 4    (h) 11    (i) 1
2. (a)  $\frac{4}{7}$     (b)  $\frac{7}{9}$     (c)  $\frac{1}{2}$   
 (d)  $\frac{4}{9}$     (e)  $\frac{1}{2}$     (f)  $\frac{3}{4}$
3. (a)  $\frac{4}{9}$     (b)  $\frac{7}{15}$     (c)  $\frac{1}{15}$   
 (d)  $\frac{11}{24}$     (e)  $\frac{7}{12}$     (f)  $\frac{1}{6}$
4. (a)  $\frac{3}{3}$     (b)  $\frac{3}{5}$     (c)  $\frac{5}{2}$   
 (d)  $\frac{4}{6}$     (e) 4    (f)  $\frac{7}{5}$
5. (a)  $2\frac{1}{3}$     (b)  $2\frac{1}{4}$     (c)  $1\frac{3}{5}$   
 (d)  $\frac{3}{4}$     (e)  $\frac{3}{11}$     (f)  $\frac{4}{7}$
6. (a)  $\frac{3}{10}$     (b)  $\frac{21}{40}$     (c) 9  
 (d)  $\frac{6}{11}$     (e)  $\frac{2}{5}$     (f)  $\frac{1}{2}$
7. (a)  $\frac{15}{28}$     (b)  $\frac{10}{33}$     (c)  $\frac{6}{7}$   
 (d)  $\frac{10}{13}$     (e) 3    (f)  $4\frac{1}{2}$   
 (g)  $\frac{1}{12}$     (h)  $\frac{1}{14}$
8. (a)  $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}$     (b)  $\frac{3}{4}, \frac{13}{16}, \frac{7}{8}$   
 (c)  $\frac{3}{5}, \frac{3}{4}, \frac{9}{10}$     (d)  $\frac{2}{3}, \frac{5}{6}, \frac{8}{9}$

## Number Sense and Numeracy: Ratio and Rate

Quantities are often compared in mathematics. A comparison of one number to another is a ratio.

### Example 1

Write as a ratio in lowest terms:  
15 basketball players to 18 football players

#### Solution

Write the ratio in symbols.

$$\underline{15 : 18} = 5 : 6$$

[15 is compared to 18.

### Example 2

If 2 people earn a total of \$23, how many people will earn a total of \$92?

#### Solution

Write the ratio in fraction form and find the missing term.

$$\frac{2}{23} = \frac{\blacksquare}{92}$$

$$\text{Since } \frac{2}{23} = \frac{2 \times 4}{23 \times 4} = \frac{8}{92}$$

Then  $\blacksquare = 8$ .

Therefore, 8 people will earn a total of \$92.

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### Practise

- Express each as a ratio.
  - 3 pucks to 7 sticks
  - 5 pens to 6 pencils
  - 7 boys to 3 girls
  - 7 foxes to 4 coyotes
  - 6 perch to 5 pike
  - 4 bears to 7 cougars
- Write each ratio in lowest terms.
  - 4 : 8
  - 6 : 18
  - 8 : 20
  - 12 : 42
  - $\frac{15}{25}$
  - $\frac{30}{42}$
  - 2 : 4 : 6
  - 18 : 6 : 24
  - 16 : 12 : 20
  - 24 : 18 : 30
- Write each ratio in lowest terms.
  - 4 fries to 7 hamburgers
  - 4 defence to 6 forwards
  - 12 min to 30 min
  - 8 horses to 12 pigs
- Write each ratio. Use the same units.

Example: 25 cm to 1 m  
Write: 25 cm to 100 cm  
The ratio is 25 : 100, or 1 : 4 in lowest terms.

  - 7 mm to 3 cm
  - 17 s to 1 min
  - 25 m to 5 cm
  - 15 s to 1 min
- Write the missing term(s) for the following.
  - 2 : 5 =  $\blacksquare$  : 10
  - 3 : 7 =  $\blacksquare$  : 21
  - $\frac{4}{7} = \frac{8}{\blacksquare}$
  - $\frac{5}{8} = \frac{15}{\blacksquare}$
  - 1 :  $\blacksquare$  : 4 = 2 : 6 :  $\blacksquare$
  - 1 : 6 :  $\blacksquare$  = 3 :  $\blacksquare$  : 9
- Express as a rate.
  - 4 tins for \$2
  - \$75 for 8 h work
  - \$4 for 3 novels
  - 79 km in 4 h
  - 3 goals for 4 shots
- Write each rate as a ratio in lowest terms.
  - 5 stamps for 85¢
  - 75¢ for 3 L of gasoline
  - 2 records for \$10.98
  - \$1.23 for 3 rolls of tape

#### Number Sense and Numeracy: Ratio and Rate

- 3 : 7
  - 5 : 6
  - 7 : 3
  - 7 : 4
  - 6 : 5
  - 4 : 7
- 1 : 2
  - 1 : 3
  - 2 : 5
  - 2 : 7
  - 3 : 5
  - 5 : 7
  - 1 : 2 : 3
  - 3 : 1 : 4
  - 4 : 3 : 5
  - 4 : 3 : 5
- 4 : 7
  - 2 : 3
  - 2 : 5
  - 2 : 3
- 7 : 30
  - 17 : 60
  - 500 : 1
  - 1 : 4
- 4
  - 9
  - 14
  - 24
  - 3, 8
  - 3, 18
- 2 tins/dollar
  - \$9.38/h
  - \$1.33/novel
  - 19.75 km/h
  - 0.75 goals/shot
- 1 : 17
  - 25 : 1
  - 1 : 5.49
  - 0.41 : 1

## Number Sense and Numeracy: Percent

There are three types of percent problems.

### Example 1

Find 20% of 65.

#### Solution

$$\begin{aligned}20\% \text{ of } 65 &= 0.20 \times 65 \\ &= 13\end{aligned}$$

Thus, 20% of 65 is 13.

### Example 2

What percent is 38 of 95?

#### Solution

$$\begin{aligned}\frac{n}{100} &= \frac{38}{95} \quad \left[ \frac{38}{95} = \frac{2}{5} = \frac{40}{100} \right] \\ \frac{n}{100} &= \frac{40}{100} \\ n &= 40\end{aligned}$$

Thus, 38 is 40% of 95.

### Example 3

60% of a number is 15.

What is the number?

#### Solution

60% of a number is 15.

1% of a number is  $\frac{15}{60}$ .

100% of a number is  $\frac{15}{60} \times 100$ .

Thus, the number is 25.

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### Practise

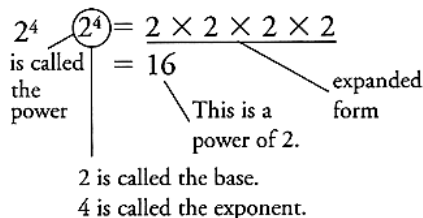
- Write each percent as a fraction in lowest terms.  
(a) 49%      (b) 75%      (c) 1%  
(d)  $\frac{1}{2}\%$       (e)  $33\frac{1}{3}\%$       (f)  $7\frac{1}{2}\%$
- Write each fraction as a percent.  
(a)  $\frac{73}{100}$       (b)  $\frac{3}{10}$       (c)  $\frac{7}{50}$   
(d)  $\frac{1}{4}$       (e)  $\frac{5}{8}$       (f) 1
- Write each decimal as a percent.  
(a) 0.43      (b) 0.92  
(c) 0.225      (d) 1.07  
(e) 3.51      (f) 0.005
- Calculate each of the following to one decimal place.  
(a) 15% of 75      (b) 75% of 68  
(c) 150% of 60      (d)  $\frac{1}{2}\%$  of 244  
(e)  $1\frac{1}{2}\%$  of 76      (f)  $2\frac{3}{4}\%$  of 748
- What percent is:  
(a) 73 of 100?      (b) 87 of 100?  
(c) 19 of 50?      (d) 13 of 25?  
(e) 13 of 10?      (f) 63 of 50?
- Find the number:  
(a) three to the exponent 2  
(b) four to the exponent 3  
(c) two to the exponent 5  
(d) five to the exponent 1  
(e) six to the exponent 0  
(f) two to the exponent 16
- Find the interest on \$1500 for one year at the following rates of interest.  
(a) 12%      (b) 8.5%  
(c)  $\frac{1}{2}\%$       (d)  $\frac{1}{4}\%$   
(e) 3%      (f)  $6\frac{1}{4}\%$
- Write the rate of discount to one decimal place on a purchase of \$150 if the discount is:  
(a) \$15      (b) \$30      (c) \$22.50  
(d) \$50      (e) \$67.50      (f) \$18.75

### Number Sense and Numeracy: Percent

- (a)  $\frac{49}{100}$       (b)  $\frac{3}{4}$       (c)  $\frac{1}{100}$   
(d)  $\frac{1}{200}$       (e)  $\frac{1}{3}$       (f)  $\frac{3}{40}$
- (a) 73%      (b) 30%      (c) 14%      5. (a) 73%      (b) 87%      (c) 38%  
(d) 25%      (e) 62.5%      (f) 100%      6. (a) 9      (b) 64      (e) 126%  
(d) 5      (e) 1      (f) 65 536
- (a) 43%      (b) 92%      (c) 22.5%      7. (a) \$180      (b) \$127.50      (c) \$7.50  
(d) 107%      (e) 351%      (f) 0.5%      (d) \$3.75      (e) \$45      (f) \$93.75
- (a) 11.3      (b) 51      (c) 90      8. (a) 10%      (b) 20%      (c) 15%  
(d) 1.2      (e) 1.1      (f) 20.6      (d) 33.3%      (e) 45%      (f) 12.5%

# Number Sense and Numeracy: Factors and Exponents

## Example 1



## Example 2

Calculate  $2^3 \times 2^2$ .

### Solution

$$2^3 \times 2^2 = 2 \times 2 \times 2 \times 2 \times 2$$

$\setminus \quad /$   
 like bases = 32

## Example 3

Evaluate  $2^2 \times 3^2$ .

### Solution

$$2^2 \times 3^2 = 2 \times 2 \times 3 \times 3$$

$\setminus \quad /$   
 unlike bases = 36

## Practise

1. Write the value of each.

- |           |           |           |
|-----------|-----------|-----------|
| (a) $2^2$ | (b) $2^3$ | (c) $2^4$ |
| (d) $3^2$ | (e) $3^3$ | (f) $3^4$ |
| (g) $4^2$ | (h) $4^3$ | (i) $5^3$ |

2. Simplify.

- |                      |                      |
|----------------------|----------------------|
| (a) $3 \times 2^2$   | (b) $3 \times 2^3$   |
| (c) $2 \times 4^2$   | (d) $2 \times 4^3$   |
| (e) $2^2 \times 3^2$ | (f) $2^2 \times 3^3$ |
| (g) $3^2 \times 2^3$ | (h) $2^2 \times 4^2$ |
| (i) $2^3 \times 4^2$ | (j) $5^2 \times 3^2$ |

3. For each power, what is the base? the exponent?

- |           |           |           |
|-----------|-----------|-----------|
| (a) $2^3$ | (b) $3^2$ | (c) $2^4$ |
| (d) $3^4$ | (e) $5^2$ |           |

4. Write each expression as a power.

- (a)  $3 \times 3 \times 3 \times 3 \times 3$   
 (b)  $2 \times 2 \times 2 \times 2$   
 (c)  $5 \times 5 \times 5$   
 (d)  $4 \times 4 \times 4 \times 4 \times 4 \times 4$

5. Write in expanded form.

- |              |           |              |
|--------------|-----------|--------------|
| (a) $5^2$    | (b) $2^5$ | (c) $6^4$    |
| (d) $x^3$    | (e) $y^4$ | (f) $(2m)^3$ |
| (g) $(3n)^2$ | (h) $4^3$ | (i) $3^4$    |

6. Evaluate:

- (a) the third power of 2  
 (b) the fourth power of 3  
 (c) the second power of 5  
 (d) the fifth power of 1  
 (e) the sixth power of 0  
 (f) the second power of 16

7. Write each number as a power of 10.

- |             |                |
|-------------|----------------|
| (a) 100     | (b) 1000       |
| (c) 100 000 | (d) 1 000 000  |
| (e) 10      | (f) 10 000 000 |

8. Write as a power of 2.

- |        |         |
|--------|---------|
| (a) 4  | (b) 16  |
| (c) 64 | (d) 256 |

9. Evaluate.

- |                    |                      |
|--------------------|----------------------|
| (a) $7^2 + 2^2$    | (b) $4^3 - 2^5$      |
| (c) $3 \times 2^3$ | (d) $3^2 \times 2^2$ |

10. Evaluate for  $x = 2$ .

- |            |                |                 |
|------------|----------------|-----------------|
| (a) $2x$   | (b) $x^2$      | (c) $x^3$       |
| (d) $3x^2$ | (e) $2x^2 + 3$ | (f) $x^3 - x^2$ |

### Number Sense and Numeracy: Factors and Exponents

1. (a) 4 (b) 8 (c) 16  
 (d) 9 (e) 27 (f) 81  
 (g) 16 (h) 64 (i) 125  
 2. (a) 12 (b) 24 (c) 32 (d) 128  
 (e) 36 (f) 108 (g) 72 (h) 64  
 (i) 128 (j) 225

3. (a) base 2, exponent 3 (b) base 3, exponent 2  
 (c) base 2, exponent 4 (d) base 3, exponent 4  
 (e) base 5, exponent 2  
 4. (a)  $3^5$  (b)  $2^4$  (c)  $5^3$  (d)  $4^6$   
 5. (a)  $5 \times 5$  (b)  $2 \times 2 \times 2 \times 2 \times 2$   
 (c)  $6 \times 6 \times 6 \times 6$  (d)  $x \times x \times x$   
 (e)  $y \times y \times y \times y$  (f)  $2m \times 2m \times 2m$   
 (g)  $3n \times 3n$  (h)  $4 \times 4 \times 4$   
 (i)  $3 \times 3 \times 3 \times 3$

6. (a) 8 (b) 81 (c) 25  
 (d) 1 (e) 0 (f) 256  
 7. (a)  $10^2$  (b)  $10^3$  (c)  $10^5$   
 (d)  $10^6$  (e)  $10^1$  (f)  $10^7$   
 8. (a)  $2^2$  (b)  $2^4$  (c)  $2^6$  (d)  $2^8$   
 9. (a) 53 (b) 32 (c) 24 (d) 36  
 10. (a) 4 (b) 4 (c) 8  
 (d) 12 (e) 11 (f) 4

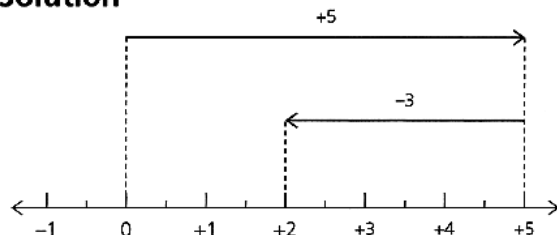
## Number Sense and Numeracy: Integers—Addition and Subtraction

A number line can be used to show how to add integers.

### Example 1

Find  $(+5) + (-3)$ .

#### Solution



$(+5) + (-3) = 2$  — The positive sign is often not used.

To subtract an integer you add its opposite.

### Example 2

Subtract: (a)  $5 - (-2)$  (b)  $-8 - (+3)$

#### Solution

$$(a) \quad 5 - (-2) = 5 + (+2) \\ = 7$$

$$(b) \quad -8 - (+3) = -8 + (-3) \\ = -11$$

To subtract an integer, you add its opposite.

### Practise

#### 1. Find each sum.

- (a)  $-3 + (-2)$       (b)  $2 + (-3)$   
 (c)  $-8 + (+8)$       (d)  $-6 + (+4)$   
 (e)  $-4 + (-5)$       (f)  $2 + (-6)$

#### 2. Add.

- (a)  $\begin{array}{r} 5 \\ -7 \\ \hline \end{array}$       (b)  $\begin{array}{r} -3 \\ -4 \\ \hline \end{array}$       (c)  $\begin{array}{r} -9 \\ -2 \\ \hline \end{array}$   
 (d)  $\begin{array}{r} -4 \\ -6 \\ \hline \end{array}$       (e)  $\begin{array}{r} 7 \\ -7 \\ \hline \end{array}$       (f)  $\begin{array}{r} 7 \\ -2 \\ \hline \end{array}$   
 (g)  $\begin{array}{r} -8 \\ -4 \\ \hline \end{array}$       (h)  $\begin{array}{r} -4 \\ -3 \\ \hline \end{array}$       (i)  $\begin{array}{r} -5 \\ -5 \\ \hline \end{array}$

#### 3. Find each difference.

- (a)  $4 - (-3)$       (b)  $-5 - (-2)$   
 (c)  $5 - (-3)$       (d)  $-4 - (-7)$   
 (e)  $6 - (-6)$       (f)  $4 - (4)$   
 (g)  $-7 - (-3)$       (h)  $-7 - (-9)$

#### 4. Subtract.

- (a)  $\begin{array}{r} -4 \\ -2 \\ \hline \end{array}$       (b)  $\begin{array}{r} 5 \\ -3 \\ \hline \end{array}$       (c)  $\begin{array}{r} -7 \\ -3 \\ \hline \end{array}$   
 (d)  $\begin{array}{r} -5 \\ -5 \\ \hline \end{array}$       (e)  $\begin{array}{r} -7 \\ -8 \\ \hline \end{array}$       (f)  $\begin{array}{r} 7 \\ -3 \\ \hline \end{array}$   
 (g)  $\begin{array}{r} -7 \\ -3 \\ \hline \end{array}$       (h)  $\begin{array}{r} -7 \\ -3 \\ \hline \end{array}$

#### 5. Simplify.

- (a)  $3 - (-4)$       (b)  $-7 + 2$   
 (c)  $5 - 3$       (d)  $3 - 5$   
 (e)  $-4 - (-4)$       (f)  $-4 - 4$   
 (g)  $5 - (-3) + 4$       (h)  $-4 - (-3) + 5$   
 (i)  $-6 - 4 - 3$       (j)  $-4 + 7 - 5$

#### 6. Which choice would make each statement true: $>$ , $<$ , or $=$ ?

- (a)  $-3 - 4 - 5 + 3$  ■  $-4 - 3 - 1 - (-2)$   
 (b)  $4 - 7 + 6 - 8$  ■  $-3 - 5 - (-7) - 4$   
 (c)  $9 - 6 - (-4) - 5$  ■  $5 - 13 - 7 - (-8)$

### Number Sense and Numeracy: Integers—Addition and Subtraction

1. (a)  $-5$       (b)  $-1$       (c)  $0$   
 (d)  $-2$       (e)  $-9$       (f)  $-4$   
 2. (a)  $-2$  (b)  $1$  (c)  $-7$  (d)  $-10$   
 (e)  $0$  (f)  $5$  (g)  $-12$  (h)  $-1$  (i)  $-10$   
 3. (a)  $7$       (b)  $-3$       (c)  $8$       (d)  $3$   
 (e)  $12$       (f)  $0$       (g)  $-4$       (h)  $2$   
 4. (a)  $-2$       (b)  $8$       (c)  $-4$       (d)  $-10$   
 (e)  $1$       (f)  $10$       (g)  $-4$       (h)  $-10$   
 5. (a)  $7$       (b)  $-5$       (c)  $2$       (d)  $-2$   
 (e)  $0$       (f)  $-8$       (g)  $12$       (h)  $4$   
 (i)  $-13$       (j)  $-2$   
 6. (a)  $<$       (b)  $=$       (c)  $>$

## Number Sense and Numeracy: Integers—Multiplication and Division

Use a pattern to remember how to multiply or divide integers.

The + and - signs are shown here to remember the rules. In the examples and practise questions, +5 is written as 5.

$$\begin{array}{cccc}
 (-5)(-2) = +10 & (-10) \div (-2) = +5 & (+5)(-2) = -10 & (-10) \div (+2) = -5 \\
 (+5)(+2) = +10 & (+10) \div (+2) = +5 & (-5)(+2) = -10 & (+10) \div (-2) = -5 \\
 \begin{array}{c} \diagdown \quad / \\ \text{same signs} \end{array} & \begin{array}{c} | \\ \text{positive integer} \end{array} & \begin{array}{c} \diagdown \quad / \\ \text{different signs} \end{array} & \begin{array}{c} | \\ \text{negative integer} \end{array}
 \end{array}$$

### Example 1

Find  $3(-2)$ .

#### Solution

$$3(-2) = -6$$

### Example 2

Calculate  $(-2)^3$ .

#### Solution

$$\begin{aligned}
 (-2)^3 &= (-2)(-2)(-2) \\
 &= -8
 \end{aligned}$$

### Example 3

Find  $(-18) \div (-3)$ .

#### Solution

$$(-18) \div (-3) = 6$$

## Practise

1. Find each product.

$$\begin{array}{ll}
 \text{(a)} (-3)(2) & \text{(b)} (-4)(-9) \\
 \text{(c)} (4)(-3) & \text{(d)} (-7)(-3) \\
 \text{(e)} (5)(4) & \text{(f)} (-2)(7)
 \end{array}$$

2. Simplify.

$$\begin{array}{lll}
 \text{(a)} -2(-7) & \text{(b)} -3(8) & \text{(c)} 5(-7) \\
 \text{(d)} -5(-7) & \text{(e)} -4(-9) & \text{(f)} -4(9)
 \end{array}$$

3. Find each quotient.

$$\begin{array}{ll}
 \text{(a)} -18 \div (-6) & \text{(b)} -24 \div 6 \\
 \text{(c)} 51 \div (-17) & \text{(d)} -42 \div (-14) \\
 \text{(e)} -18 \div (18) & \text{(f)} -24 \div (-6) \\
 \text{(g)} 60 \div (-12) & \text{(h)} -30 \div (-15)
 \end{array}$$

4. Simplify.

$$\begin{array}{lll}
 \text{(a)} \frac{-50}{5} & \text{(b)} \frac{-15}{-5} & \text{(c)} \frac{30}{-6} \\
 \text{(d)} \frac{48}{-6} & \text{(e)} \frac{16}{-16} & \text{(f)} \frac{-16}{-8} \\
 \text{(g)} \frac{18}{-9} & \text{(h)} \frac{-81}{27} & \text{(i)} \frac{-18}{-9}
 \end{array}$$

5. Evaluate.

$$\begin{array}{lll}
 \text{(a)} (-4)^2 & \text{(b)} (-2)^4 & \text{(c)} (-3)^4 \\
 \text{(d)} (-5)^2 & \text{(e)} -5^2 & \text{(f)} 4^3
 \end{array}$$

6. Simplify.

#### Example:

$$\begin{aligned}
 -3(-2)^4 &= -3(16) && \left[ \begin{array}{l} \text{Calculate} \\ \text{powers first.} \end{array} \right. \\
 &= -48
 \end{aligned}$$

$$\begin{array}{ll}
 \text{(a)} -2(-3)^2 & \text{(b)} 4(-2)^3 \\
 \text{(c)} 5(-3)^3 & \text{(d)} (-3)^2(-2)^2 \\
 \text{(e)} -3^2(-2)^3 & \text{(f)} (5)^2(-2)^2 \\
 \text{(g)} -5^2(-3) & \text{(h)} (-5)^2(-3)
 \end{array}$$

7. Calculate.

$$\begin{array}{ll}
 \text{(a)} (-6)^2 \div (-3) & \text{(b)} -6^3 \div (-3) \\
 \text{(c)} -3^4 \div (-3)^2 & \text{(d)} (-4)^3 \div (-2)^3 \\
 \text{(e)} 6^2 \div (-3)^2 & \text{(f)} -4^2 \div (-2)^3 \\
 \text{(g)} 2(-4)^2 \div (-8) & \text{(h)} -8 \div [(-2)(4)] \\
 \text{(i)} -8(-3) \div (-2)^2
 \end{array}$$

### Number Sense and Numeracy: Integers—Multiplication and Division

- |           |         |         |            |         |        |            |            |         |          |         |
|-----------|---------|---------|------------|---------|--------|------------|------------|---------|----------|---------|
| 1. (a) -6 | (b) 36  | (c) -12 | 3. (a) 3   | (b) -4  | (c) -3 | (d) 3      | 6. (a) -18 | (b) -32 | (c) -135 | (d) 36  |
| (d) 21    | (e) 20  | (f) -14 | (e) -1     | (f) 4   | (g) -5 | (h) 2      | (e) 72     | (f) 100 | (g) 75   | (h) -75 |
| 2. (a) 14 | (b) -24 | (c) -35 | 4. (a) -10 | (b) 3   | (c) -5 | 7. (a) -12 | (b) 72     | (c) -9  |          |         |
| (d) 35    | (e) 36  | (f) -36 | (d) -8     | (e) -1  | (f) 2  | (d) 8      | (e) 4      | (f) 2   |          |         |
|           |         |         | (g) -2     | (h) -3  | (i) 2  | (g) -4     | (h) 1      | (i) 6   |          |         |
|           |         |         | 5. (a) 16  | (b) 16  | (c) 81 |            |            |         |          |         |
|           |         |         | (d) 25     | (e) -25 | (f) 64 |            |            |         |          |         |

## Number Sense and Numeracy: Order of Operations

When calculating expressions, we follow the order of operations:

Order of Operations

1. Simplify brackets.
2. Then simplify powers.
3. Multiply and divide.
4. Then add and subtract.

### Example 1

Simplify

$$-3(2 - 4) - (-2 + 4).$$

**Solution**

$$\begin{aligned} & -3(2 - 4) - (-2 + 4) \\ & = -3(-2) - (2) \\ & = 6 - 2 \\ & = 4 \end{aligned}$$

### Example 2

Add  $(-3)^2$  to the product of  $-2$  and  $4$ .

**Solution**

$$\begin{aligned} & (-2)(4) + (-3)^2 \\ & = (-2)(4) + 9 \\ & = -8 + 9 \\ & = 1 \end{aligned}$$

---

### Practise

1. Simplify. Use the order of operations.

- (a)  $5 - (3 - 4)$
- (b)  $(5 - 7) - (3 - 4)$
- (c)  $-3(-4) - (5 - 7)$
- (d)  $(3)(2) - (3 + 5)$
- (e)  $-(5 - 9) - (-2)(2)$
- (f)  $(4 - 3) - 2(3 - 4)$
- (g)  $4(-2) - (-8 + 4)$

2. Simplify.

- (a)  $2(-3)^2 - 4(-2)$
- (b)  $-4(-2)^3 - 3(-4)^2$
- (c)  $(-3 - 2)^2 - (2 + 4)^2$
- (d)  $3(-2 + 4)^3 - 2(-4 + 1)^2$
- (e)  $2(-1 - 3)^2 - (1 + 3)^2$
- (f)  $5(-2)^2 - 3(-1 - 2)^3$
- (g)  $3(-1 - 2) - (5 - 7)^3$
- (h)  $5(-2 + 1)^3 - (-3 - 2)^2$
- (i)  $4(1 - 3)^3 - (4 - 7)^2$
- (j)  $-2(1 - 4)^3 - 3(1 - 5)^2$

3. Match each expression with the corresponding sentence.

- (a) Find the sum of  $-3$ ,  $-8$ , and  $-2$ .
- (b)  $-8$  is added to the product of  $-3$  and  $-2$ .
- (c) Subtract  $-8$  from the product of  $-3$  and  $-2$ .
- (d) Divide  $-8$  by  $-2$  and add  $-3$ .
- (e) Decrease the quotient of  $-8$  and  $-2$  by  $-3$ .
  - i.  $-3(-2) + (-8)$
  - ii.  $-8 \div (-2) + (-3)$
  - iii.  $-8 \div (-2) - (-3)$
  - iv.  $-8 + (-3) + (-2)$
  - v.  $(-3)(-2) - (-8)$

4. Increase the sum of  $-3$  and  $5$  by  $-6$ .

5. Divide the sum of  $7$  and  $-16$  by  $-3$ .

6. By how much is the sum of  $-8$  and  $6$  more than  $-4$ ?

7. How much less is the sum of  $-8$  and  $-4$  than the product of  $4$  and  $-2$ ?

8. Divide  $-4^2$  by  $(-2)^2$ .

9. Add  $-3^2$  and  $4^2$  to the product of  $6$  and  $-2$ .

10. Increase the product of  $-7$  and  $-3$  by  $-18$ .

11. By how much is  $(-3 + 5)^2$  more than  $-(-3 + 5)^2$ ?

### Number Sense and Numeracy: Order of Operations

1. (a) 6 (b)  $-1$  (c) 14 (d)  $-2$   
(e) 8 (f) 3 (g)  $-4$
2. (a) 26 (b)  $-16$  (c)  $-11$  (d) 6  
(e) 16 (f) 101 (g)  $-1$  (h)  $-30$   
(i)  $-41$  (j) 6
3. (a) iv. (b) i. (c) v.  
(d) ii. (e) iii.
4.  $-4$  5. 3 6. 2 7. 4
8.  $-4$  9.  $-5$  10. 3 11. 8

## Number Sense and Numeracy: Square Roots

A square has an area of  $25 \text{ m}^2$ . Find the length of each side.

$A = l \times w$  In a square,  $l = w$ , so

$$A = l \times l$$

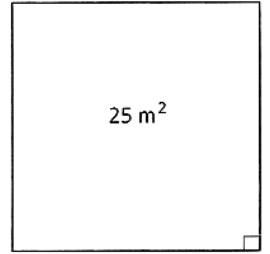
$$A = l^2$$

$$25 = l^2$$

What number multiplied by itself equals 25?

$$5 \times 5 = 25$$

$$\therefore l = 5 \text{ m}$$



The **square root** of a number is the number that multiplies by itself to give a required value. The square root symbol is  $\sqrt{\quad}$ . For instance, the square root of 25 is written as  $\sqrt{25}$ .

$$\sqrt{36} = 6, \text{ since } 6 \times 6 = 36 \quad \sqrt{0.25} = 0.5, \text{ since } 0.5 \times 0.5 = 0.25$$

### Example 1

Is the square root of 17 closer to 5 or to 4? Do not use technology to find the square root.

#### Solution

$$5 \times 5 = 25 \text{ and} \\ 4 \times 4 = 16$$

17 is closer to 16 than to 25, so  $\sqrt{17}$  is closer to 4.

### Example 2

Which is the better estimate of the square root of 50: 7.1 or 7.5?

#### Solution

$\sqrt{49} = 7$ , so 7.1 is the better estimate.

### Example 3

Is the square root of 35 closer to 5 or to 6? Do not use technology to find the square root.

#### Solution

$$5 \times 5 = 25 \text{ and} \\ 6 \times 6 = 36$$

35 is closer to 36 than to 25, so  $\sqrt{35}$  is closer to 6.

### Example 4

If  $\sqrt{121} = 11$ , what is  $\sqrt{1.21}$ ?

#### Solution

$$1.1 \times 1.1 = 1.21, \\ \text{so } \sqrt{1.21} = 1.1$$

## Practise

1. Select the best answer from those given.

Do not use a calculator.

(a)  $\sqrt{27}$  4.8, 5.2, 6.3

(b)  $\sqrt{41}$  5.9, 6.4, 7.1

(c)  $\sqrt{92}$  9.6, 10.1, 11.5

2. Select the better answer from those given.

Do not use a calculator.

(a)  $\sqrt{34}$  5.2, 5.9

(b)  $\sqrt{8}$  2.3, 2.8

(c)  $\sqrt{50}$  7.1, 7.8

3. State the value of each square root.

(a)  $\sqrt{0.09}$

(e)  $\sqrt{0.81}$

(b)  $\sqrt{0.36}$

(f)  $\sqrt{1.69}$

(c)  $\sqrt{1.44}$

(g)  $\sqrt{0.64}$

(d)  $\sqrt{0.25}$

(h)  $\sqrt{1.96}$

4. Determine all of the prime factors of each number. Use these to write the square root of the number.

(a) 225

(b) 324

(c) 484

### Number Sense and Numeracy: Square Roots

1. (a) 5.2

(b) 6.4

(c) 9.6

3. (a) 0.3

(b) 0.6

(c) 1.2

(d) 0.5

2. (a) 5.9

(b) 2.8

(c) 7.1

4. (a) 15

(b) 18

(c) 0.8

(d) 1.4

(b) 18

(c) 22



## Patterning and Algebra: Variables and Substitution

To simplify expressions, operations must be performed in a certain order. The order of operations is:

1. Simplify calculations in brackets.
2. Calculate powers.
3. Do  $\times$  and  $\div$  in the order in which they appear.
4. Do  $+$  and  $-$  in the order in which they appear.

### Example 1

Simplify  $8 \div 2 + 16 \div 4$ .

#### Solution

$$\begin{array}{l} 8 \div 2 + 16 \div 4 = 4 + 4 \\ \qquad \qquad \qquad = 8 \end{array}$$

Do multiplication or division before adding or subtracting.

### Example 2

Simplify  $(3 + 2) \times 3^2$ .

#### Solution

$$\begin{array}{l} (3 + 2) \times 3^2 = 5 \times 3^2 \\ \qquad \qquad \qquad = 5 \times 9 \\ \qquad \qquad \qquad = 45 \end{array}$$

Do the calculations in brackets first.

### Example 3

Evaluate  $4k - 3$  if  $k = 2$ .

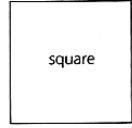
#### Solution

$$\begin{array}{l} 4k - 3 = 4(2) - 3 \\ \qquad \qquad = 8 - 3 \\ \qquad \qquad = 5 \end{array}$$

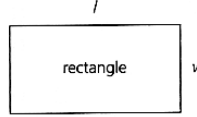
Substitute 2 for  $k$ .

## Practise

1. Use the order of operations to simplify each of the following.
  - (a)  $3 \times 5 + 6$
  - (b)  $18 \div 3 - 4$
  - (c)  $2 \times 8 + 16 \div 4$
  - (d)  $36 \div 3 - 2 \times 6$
  - (e)  $16 - 8 \div 4 \times 2$
  - (f)  $(3 \times 4 + 2) \div 7$
  - (g)  $(8 + 4) \times 6 \div 3$
2. Simplify the following.
  - (a)  $3^3 - 2^2$
  - (b)  $3 + 2^3$
  - (c)  $\frac{5 + 8 \times 2}{4 + 3}$
  - (d)  $\frac{(3 + 4) \times 6}{3 \times 4 + 2}$
  - (e)  $\frac{5 \times 2 + 7 \times 8}{3 \times 2 + 15 \div 3}$
  - (f)  $\frac{4 \times (17 - 5)}{1 + 45 \div 3}$
3. Find the value of each expression if  $k = 3$ .
  - (a)  $5(k - 1)$
  - (b)  $5k - 1$
  - (c)  $5k^2 - 1$
  - (d)  $5(k + 1)$
4. Evaluate each expression if  $k = 3$ ,  $h = 2$ .
  - (a)  $7k + h$
  - (b)  $2h + k$
  - (c)  $3(k + h)$
  - (d)  $12(k - h)$
  - (e)  $k^2 + h$
  - (f)  $h^2 + k$
  - (g)  $3(k + h)^2$
  - (h)  $5(k - h)^3$
5. (a) Evaluate each expression if  $a = 3$ ,  $b = 2$ .
  - $4a^2b^2$
  - $(2ab)^2$
 (b) Comment on the results.
6. Match the figure to the formula.
 

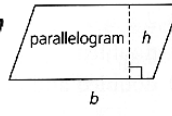


square

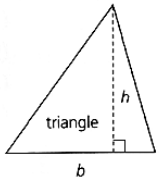


rectangle

  - (a)  $A = l \times w$
  - (b)  $A = \frac{1}{2} \times b \times h$
  - (c)  $A = b \times h$
  - (d)  $A = s^2$



parallelogram



triangle
7. Use the formulas in Question 6. Find the area of the following figures. Round your answers to one decimal place.
  - (a) rectangle:  $l = 8$  cm,  $w = 3$  cm
  - (b) triangle:  $h = 5.3$  cm,  $b = 7.2$  cm
  - (c) square:  $s = 9.3$  cm
  - (d) parallelogram:  $b = 9.4$  cm,  $h = 5.2$  cm

### Patterning and Algebra: Variables and Substitution

1. (a) 21 (b) 2 (c) 20 (d) 0  
(e) 12 (f) 2 (g) 8
2. (a) 23 (b) 11 (c) 3  
(d) 3 (e) 6 (f) 3
3. (a) 10 (b) 14 (c) 44 (d) 20
4. (a) 23 (b) 7 (c) 15 (d) 12  
(e) 11 (f) 7 (g) 75 (h) 5
5. (a) 144, 144 (b) same
6. (a) rectangle:  $A = l \times w$ ; (d) square:  $A = s^2$   
(b) triangle:  $A = \frac{1}{2} \times b \times h$  (c) parallelogram:  $A = b \times h$
7. (a)  $24 \text{ cm}^2$  (b)  $19.1 \text{ cm}^2$  (c)  $86.5 \text{ cm}^2$  (d)  $48.9 \text{ cm}^2$

## Patterning and Algebra: Solving Equations

To solve equations by inspection, write equivalent equations using the operations: +, −, ×, and ÷.

### Example 1

Solve  $3n - 5 = 13$ .

#### Solution

Something minus 5 must equal 13.

$$18 - 5 = 13$$

This means that  $3n = 18$ .

Three times what gives eighteen?

$$3 \times 6 = 18$$

Therefore, the solution is  $n = 6$ .

### Example 2

Verify that  $n = 6$  is a solution of

$$3n - 5 = 13.$$

#### Solution

$$\begin{aligned} \text{L.S.} &= 3n - 5 & \text{R.S.} &= 13 \\ &= 3(6) - 5 & \leftarrow \text{Substitute 6 for } n. \\ &= 18 - 5 \\ &= 13 \end{aligned}$$

$$\text{L.S.} = \text{R.S.}$$

Therefore, 6 is a solution of the equation.

### Practise

#### 1. Solve by inspection.

(a) $n + 3 = 7$	(b) $f + 5 = 2$
(c) $9 = 3 + x$	(d) $9 + g = 3$
(e) $n - 4 = 7$	(f) $z - 2 = 13$

#### 2. Solve by inspection.

(a) $2x = 6$	(b) $3n = 18$
(c) $4c = -16$	(d) $-4m = 20$
(e) $-30 = 6b$	(f) $-25 = -5a$

#### 3. Solve by inspection.

(a) $2k + 1 = 7$	(b) $6 + 3k = 27$
(c) $18 = 4a - 2$	(d) $11 = 2 - 3y$
(e) $3 - 4z = -5$	(f) $4p - 3 = 9$
(g) $-6 = 8 - 7v$	(h) $3h - 4 = -4$
(i) $8 = 2 - 3y$	(j) $4y - 2 = 2$

#### 4. Find the solution set for each equation.

(a) $\frac{1}{3}m = 4$	(b) $\frac{3}{4}e = 15$
(c) $-6 = \frac{3}{4}b$	(d) $20 = \frac{-5}{8}a$
(e) $\frac{-5}{8}y = -30$	(f) $\frac{-3}{11}c = 0$

#### 5. Solve. Verify your answers.

(a) $\frac{1}{3}m + 1 = 3$	(b) $\frac{1}{5}k + 5 = 2$
(c) $\frac{2}{5}n - 5 = 3$	(d) $3 = \frac{3}{4}e - 18$
(e) $\frac{3}{4}h - 4 = 2$	

#### 6. Match each sentence with the correct equation.

- |  |
|--|
| (a) Three times a number is equal to twenty-one. |
| (b) A number increased by three is ten.          |
| (c) A number divided by five is twenty.          |
| (d) Jim's age five years ago was fifteen.        |

i.  $n + 3 = 10$       iii.  $3x = 21$

ii.  $a - 5 = 15$       iv.  $\frac{m}{5} = 20$

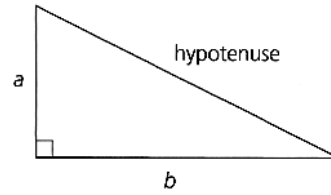
#### 7. Write an equation for each sentence, and then solve it.

- |  |
|--|
| (a) Two times a number is twenty-six.            |
| (b) Five less than a number is two.              |
| (c) A number decreased by four is negative nine. |
| (d) Three more than twice a number is 19.        |

- Patterning and Algebra: Solving Equations**
- |                |              |              |                 |               |              |                          |                          |              |
|----------------|--------------|--------------|-----------------|---------------|--------------|--------------------------|--------------------------|--------------|
| 1. (a) $n = 4$ | (b) $f = -3$ | (c) $x = 6$  | 3. (a) $k = 3$  | (b) $k = 7$   | (c) $a = 5$  | 5. (a) $m = 6$           | (b) $k = -15$            | (c) $n = 20$ |
| (d) $g = -6$   | (e) $n = 11$ | (f) $z = 15$ | (d) $y = -3$    | (e) $z = 2$   | (f) $p = 3$  | (d) $e = 28$             | (e) $h = 8$              |              |
| 2. (a) $x = 3$ | (b) $n = 6$  | (c) $c = -4$ | (g) $v = 2$     | (h) $h = 0$   | (i) $y = -2$ | 6. (a) iii.              | (b) i.                   | (c) iv.      |
| (d) $m = -5$   | (e) $h = -5$ | (f) $a = 5$  | (j) $y = 1$     | (b) $\{20\}$  | (c) $\{-8\}$ | 7. (a) $2x = 26, x = 13$ | (b) $n - 5 = 2, n = 7$   | (d) ii.      |
|                |              |              | 4. (a) $\{12\}$ | (c) $\{-32\}$ | (e) $\{48\}$ | (c) $n - 4 = -9, n = -5$ | (d) $2x + 3 = 19, x = 8$ |              |

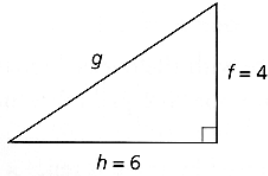
## Patterning and Algebra: Working with Right Triangles

One of the many important relationships in mathematics is known as the Pythagorean property of a right-angled triangle. The area of the square of the hypotenuse is equal to the sum of the areas of the squares of the other two sides.



### Example

Find the value of  $g$ .



### Solution

$$g^2 = f^2 + h^2$$

$$g^2 = 4^2 + 6^2$$

$$= 16 + 36$$

$$= 52$$

$$g \doteq 7.2$$

Use your calculator.

The positive square root of 52 is rounded to one decimal place.

### Practise

Round your answers to one decimal place.

1. Calculate to the number of decimal places indicated in brackets.

(a)  $\sqrt{65}$  (2)

(b)  $\sqrt{43}$  (2)

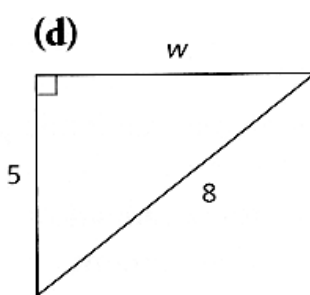
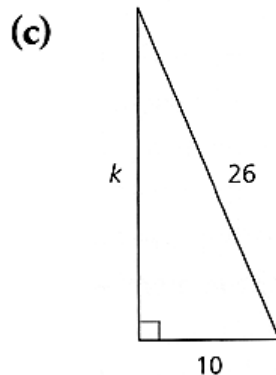
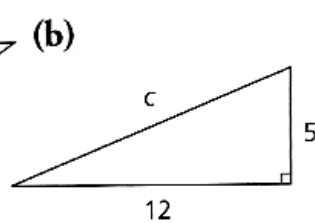
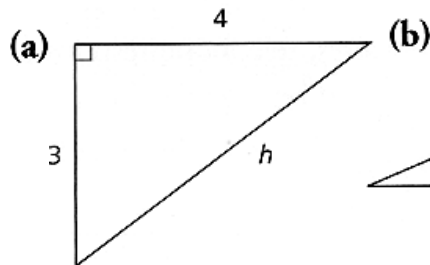
(c)  $\sqrt{359}$  (1)

(d)  $\sqrt{262}$  (1)

(e)  $\sqrt{5.52}$  (2)

(f)  $\sqrt{0.89}$  (2)

2. For each right-angled triangle, write the equation for the Pythagorean property.



3. Calculate the length of the missing side for each part of Question 2.

(a)  $h = \blacksquare$

(b)  $c = \blacksquare$

(c)  $k = \blacksquare$

(d)  $w = \blacksquare$

4. For each equation find the value of the missing measure.

(a)  $a^2 = 8^2 + 6^2$

(b)  $g^2 = 7^2 + 10^2$

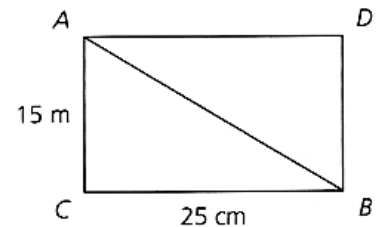
(c)  $14^2 = c^2 + 8^2$

(d)  $13^2 = n^2 + 5^2$

(e)  $16^2 = 9^2 + y^2$

(f)  $h^2 = 1.4^2 + 4^2$

5. A rectangular park is crossed from  $A$  to  $B$  using the diagonal path. How many metres are saved by not going from  $A$  to  $C$  to  $B$ ?



6. A shadow is cast by a building. The distance from the tip of the shadow to the top of the building is 85 m. The tip of the shadow is 55 m from the base of the building. How tall is the building?

### Patterning and Algebra: Working with Right Triangles

1. (a) 8.06 (b) 6.56 (c) 18.9 (d) 16.2 (e) 2.35 (f) 0.94
2. (a)  $h^2 = 4^2 + 3^2$  (b)  $c^2 = 5^2 + 12^2$  (c)  $26^2 = 10^2 + k^2$  (d)  $8^2 = 5^2 + w^2$
3. (a) 5 (b) 13 (c) 24 (d) 6.2
4. (a)  $a = 10$  (b)  $g = 12.2$  (c)  $c = 11.5$  (d)  $n = 12$  (e)  $y = 13.2$  (f)  $h = 4.2$
5. 11 m 6. 64.8 m

## Data Management: Measures of Central Tendency

Three measures of central tendency are mean, median, and mode.

### Example 1

Find the mean of these heights: 163 m, 160 m, 165 m, 165 m, 161 m

#### Solution

The mean is the average.

Mean

$$\begin{aligned} &= \frac{163 + 160 + 165 + 165 + 161}{5} \\ &= \frac{814}{5} = 162.8 \end{aligned}$$

Thus, the mean is 162.8 m.

### Example 2

Find the median for the heights in Example 1.

#### Solution

The median is the middle measurement when the numbers are arranged from the least to greatest.

160 161 163 165 165  
↑            ↑            ↑  
least        median      greatest

Thus, the median is 163 m.

### Example 3

Find the mode of the heights in Example 1.

#### Solution

The mode is the measure that occurs most frequently.

160 161 163 165 165  
                         ↓    ↓  
Thus, the mode is 165 m.

### Practise

- The mass, in kilograms, of ten people randomly chosen from a crowd is listed below.  
74 71 68 84 54  
60 64 68 68 65
  - Find the mode of the data.
  - Find the median of the data.
  - Find the mean of the data, to one decimal place.
  - If another person were randomly chosen from the same crowd, what would be your best estimate of this person's mass?
- A group of ten girls had the following ages, in years.  
16 15 15 16 14  
16 16 15 15 15
  - Find the mean, median, and mode of the data.
  - Two grandmothers, one aged 65 and the other 72, joined the group. Which measure(s) of central tendency remained unchanged?
  - Calculate the mean for the combined data of (a) and (b). (Why has it changed from your answer in (a)?)
- Students of City High had a frisbee-throwing contest. These distances, in metres, were recorded for the frisbee throws.  
45 52 47 55 62 80 71  
60 84 57 46 50 48 61  
49 63 54 63 49 72
  - Calculate the mean, median, and mode for the data, to one decimal place.
  - Why is the mode not a good measure of central tendency for the above data?

#### Data Management:

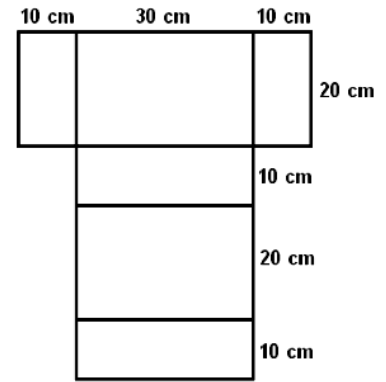
#### Measures of Central Tendency

- (a) 68 kg (b) 68 kg (c) 67.6 kg (d) 67.6 kg
- (a) mean 15.3 a, median 15 a, mode 15 a  
(b) mode (c) mean 24.2 a
- (a) mean 58.4 m, median 56 m, mode 49 m and 63 m  
(b) Two measures each occur twice.

## Surface Area and Volume

**Example 1:** A cereal box measures 30 cm by 20 cm by 10 cm. Show a net that can be used to make the box.

**Solution:** Several nets are possible. One of them is shown.



**Example 2:** Find the surface area of the box in Example 1.

**Solution:** Use the formula for the surface area of a box, or rectangular prism. The length is 30 cm, the width is 20 cm, and the height is 10 cm.

$$\begin{aligned} S &= 2lw + 2wh + 2lh \\ &= 2(30 \times 20) + 2(20 \times 10) + 2(30 \times 10) \\ &= 2200 \text{ cm}^2 \end{aligned}$$

The surface area is 2200 cm<sup>2</sup>.

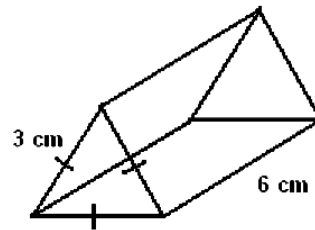
**Example 3:** Find the volume of the box in Example 1.

**Solution:** Use the formula for the volume of a box.

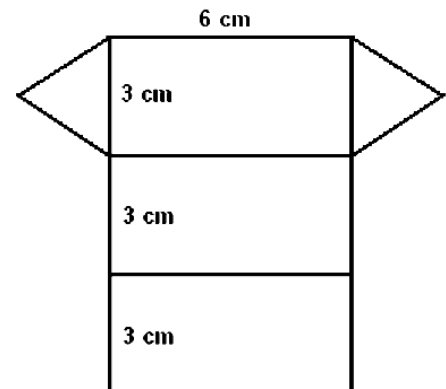
$$\begin{aligned} V &= lwh \\ &= 30 \times 20 \times 10 \\ &= 6000 \text{ cm}^3 \end{aligned}$$

The volume is 6000 cm<sup>3</sup>.

**Example 4:** A triangular prism for use in a camera is shown. Draw a net that can be used to make a model of the prism.



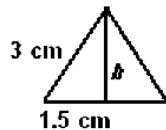
**Solution:** Several nets are possible. One of them is shown.



**Example 5:** Find the surface area and volume of the triangular prism in Example 4.

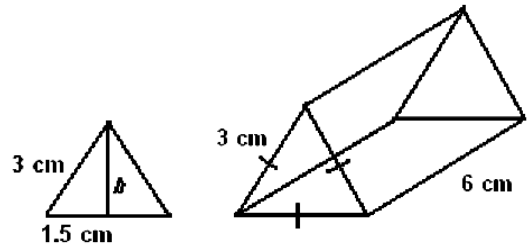
**Solution:** You will need to calculate the area of the triangular end of the prism. Use the Pythagorean theorem to find the height.

$$\begin{aligned} h^2 &= 3^2 - 1.5^2 \\ &= 6.75 \\ h &= 2.6 \text{ cm} \end{aligned}$$



The area of the triangle is calculated:

$$\begin{aligned}
 A &= \frac{1}{2}bh \\
 &= \frac{1}{2} \times (3) \times (2.6) \\
 &= 3.9 \text{ cm}^2
 \end{aligned}$$



The surface area of the prism is twice the area of the triangle plus three times the area of one side.

$$\begin{aligned}
 S &= 2 \times (3.9) + 3 \times (3) \times (6) \\
 &= 61.8 \text{ cm}^2
 \end{aligned}$$

The surface area is  $61.8 \text{ cm}^2$ .

The volume of the prism is given by the area of the triangle multiplied by the length.

$$\begin{aligned}
 V &= 3.9 \times 6 \\
 &= 23.4 \text{ cm}^3
 \end{aligned}$$

The volume is  $23.4 \text{ cm}^3$ .

### Practice:

1. A sand box measures 1.5 m by 1.8 m by 0.25 m. Sketch a net to represent the sand box.

2. Find the surface area and volume of the sand box in question 1.

3. A triangular prism is shown. Sketch a net to represent the prism.



4. Find the surface area and volume of the prism in question 3.

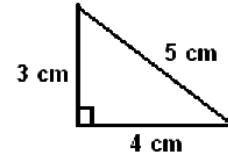
Answers:

2.  $S = 7.05 \text{ m}^2$ ,  $V = 0.675 \text{ m}^3$

4.  $S = 920 \text{ cm}^2$ ,  $V = 1200 \text{ cm}^3$

## Area and Perimeter

**Example 1:** Find the area and perimeter of the triangle shown.



**Solution:** The area of a triangle is given by  $A = \frac{1}{2}bh$ . The base of the given triangle is 4 cm, and the height is 3 cm.

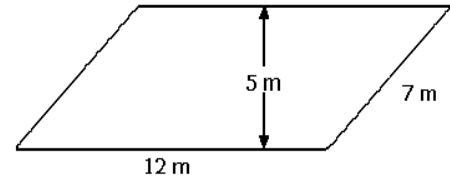
$$\begin{aligned}A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 4 \times 3 \\ &= 6 \text{ cm}^2\end{aligned}$$

The perimeter of a triangle is calculated by adding the lengths of the three sides.

$$\begin{aligned}P &= a + b + c \\ &= 3 + 4 + 5 \\ &= 12 \text{ cm}\end{aligned}$$

**Example 2:** Find the area and perimeter of the parallelogram shown.

**Solution:** The area of a parallelogram is given by  $A = bh$ . The base of the given parallelogram is 12 m, and the height is 5 m.



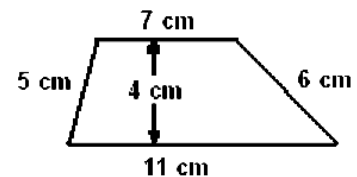
$$\begin{aligned}A &= bh \\ &= 12 \times 5 \\ &= 60 \text{ m}^2\end{aligned}$$

The perimeter of a parallelogram is calculated by adding the lengths of the four sides.

$$\begin{aligned}P &= a + b + c + d \\ &= 7 + 12 + 7 + 12 \\ &= 38 \text{ m}\end{aligned}$$

**Example 3:** Find the area and perimeter of the trapezoid shown.

**Solution:** The area of a trapezoid is given by  $A = \frac{1}{2}h(a + b)$ . In the trapezoid shown,  $a = 11$ ,  $b = 7$ , and  $h = 4$ .



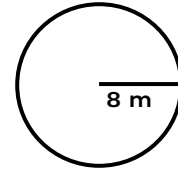
$$\begin{aligned}A &= \frac{1}{2}h(a + b) \\ &= \frac{1}{2} \times 4 \times (11 + 7) \\ &= 36 \text{ cm}^2\end{aligned}$$

The perimeter of a trapezoid is calculated by adding the lengths of the four sides.

$$\begin{aligned} P &= a + b + c + d \\ &= 5 + 7 + 6 + 11 \\ &= 29 \text{ cm} \end{aligned}$$

**Example 4:** Find the area and perimeter of the circle shown.

**Solution:** The area of a circle is given by  $A = \pi r^2$ . The radius of the circle shown is 8 m.



$$\begin{aligned} A &= \pi r^2 \\ &= \pi(8)^2 \\ &= 201.1 \text{ m}^2 \end{aligned}$$

The perimeter, or circumference, of a circle is given by  $C = 2\pi r$ .

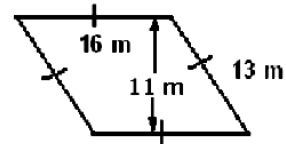
$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(8) \\ &= 50.3 \text{ m} \end{aligned}$$

**Practice:**

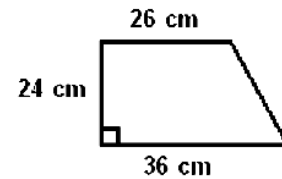
1. Find the area and perimeter of the triangle shown.



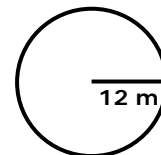
2. Find the area and perimeter of the parallelogram shown.



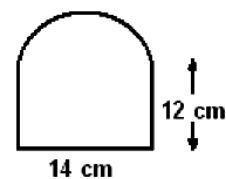
3. Find the area and perimeter of the trapezoid shown.



4. Find the area and perimeter of the circle shown.



5. Find the area and perimeter of the figure shown.

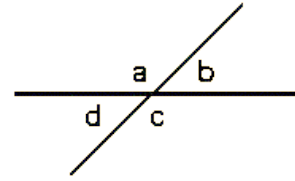


Answers: 1.  $24 \text{ cm}^2$ ,  $24 \text{ cm}$  2.  $208 \text{ m}^2$ ,  $58 \text{ m}$  3.  $744 \text{ cm}^2$ ,  $112 \text{ cm}$  4.  $452 \text{ m}^2$ ,  $75 \text{ m}$   
5.  $245 \text{ cm}^2$ ,  $60 \text{ cm}$

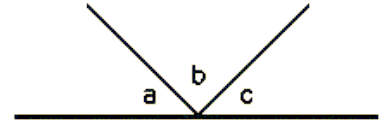


## Lines and Angles

If **two straight lines cross each other**, four angles are formed. The **sum of these angles** (a, b, c, d) will always be **360°**. The **angles vertically opposite each other** when straight lines cross will always be **equal**. In the given diagram, '**a**' equals '**c**' and '**b**' equals '**d**'.

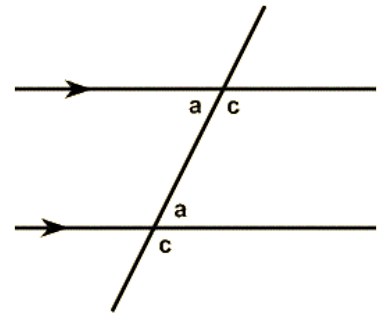


If any number of straight lines meets at the same point of another straight line, the **sum of the angles formed on a straight line** will always be **180°**. Here **the angles a, b, and c, add up to 180°**.



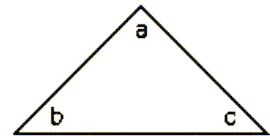
## Parallel Lines and Angles

In the given diagram, the two horizontal lines are parallel to each other, and are crossed by a sloping straight line (a transversal), causing angles to be formed. The angles marked 'c' are equal to each other, and they are called **corresponding angles**. Each 'c' angle has an **adjacent** angle marked 'a'. As you have already learnt, the sum of each pair of 'a' and 'c' angles will always be 180° (as angles on a straight line). Also the two 'a' angles are equal to each other, as **alternate angles** between the two parallel lines.

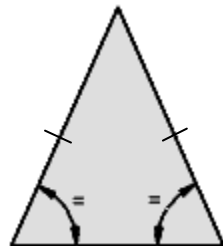


## Interior angles of a Triangle

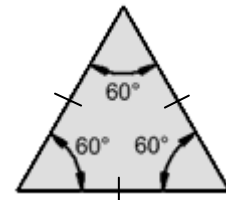
With every triangle, the sum of its three angles will always be 180°. In every triangle, sum of its angles **a + b + c = 180°**.



Scalene Triangle

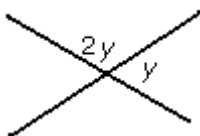


Isosceles Triangle



Equilateral Triangle

**Example 1: Find the unknown angle measures without using a protractor.**



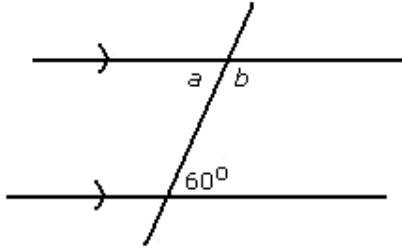
$$2y + y = 180^\circ \text{ (sum of angles on a straight line)}$$

$$3y = 180^\circ$$

$$y = 180^\circ \div 3$$

$$y = 60^\circ$$

**Example 2: Find the unknown angle measures without using a protractor.**



$$a = 60^\circ \text{ (alternate angles)}$$

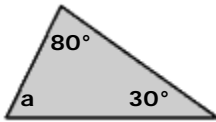
$$a + b = 180^\circ \text{ (sum of angles on a straight line)}$$

$$60^\circ + b = 180^\circ$$

$$b = 180^\circ - 60^\circ$$

$$b = 120^\circ$$

**Example 3: For the given triangle, find the measure of the unknown angle.**



$$a + 80^\circ + 30^\circ = 180^\circ \text{ (sum of angles in a triangle)}$$

$$a = 180^\circ - 80^\circ - 30^\circ$$

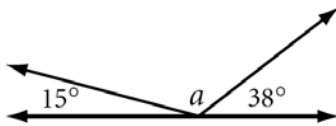
$$a = 180^\circ - 110^\circ$$

$$a = 70^\circ$$

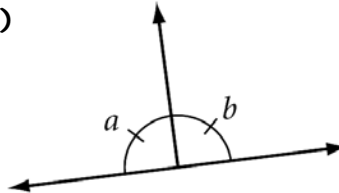
**Practice**

**1. For each given diagram, find the unknown angle measures without using a protractor.**

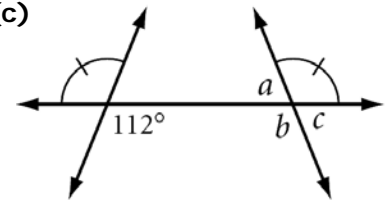
(a)



(b)

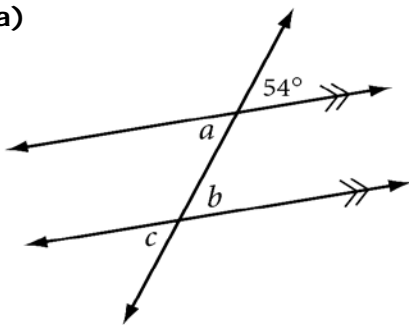


(c)

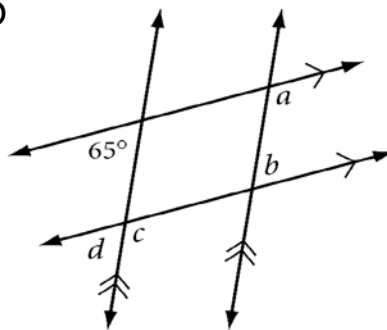


**2. For each given diagram, find the unknown angle measures without using a protractor.**

(a)

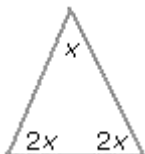


(b)

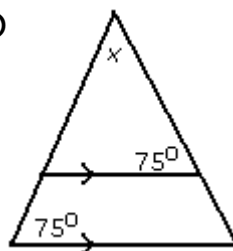


**3. Find the angle measure of "a" in each triangle.**

(a)



(b)



**Answers:**

1. (a)  $a = 127^\circ$  (b)  $a = b = 90^\circ$

(c)  $a = 68^\circ$ ,  $b = 112^\circ$ ,  $c = 68^\circ$

2. (a)  $a = 54^\circ$ ,  $b = 54^\circ$ ,  $c = 54^\circ$

(b)  $a = 115^\circ$ ,  $b = 65^\circ$ ,  $c = 115^\circ$ ,  $d = 65^\circ$

3. (a)  $x = 36^\circ$  (b)  $x = 30^\circ$